CHAPTER

2

Statistical Physics P313

Statistical Thermodynamics

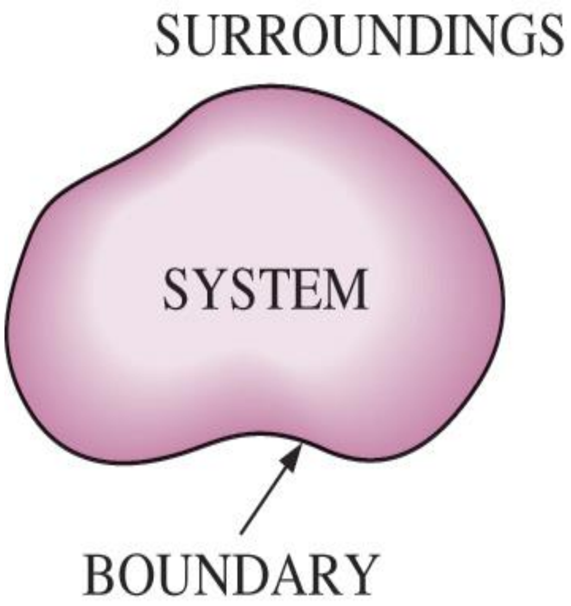
New Valley University

System, boundary and surroundings

- **System:** **A quantity of matter or a region in space chosen for study.**

- **Boundary:** **The real or imaginary surface that separates the system from its surrounding.**

- **Surrounding:** **The mass or region outside the system**



Macroscopic Viewpoint

It includes studying and analyzing system properties such as pressure *P*, volume *V*, and temperature *T*, which are properties that can be measured to give a visual description that can be seen with the naked eye.

This study is characterized by the following:

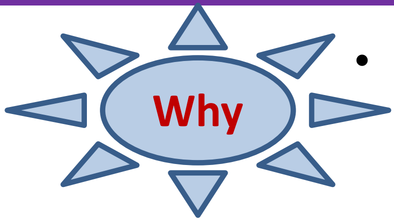
- **It does not include any assumptions about material composition.**
- **To describe the system, it suffices to know very few quantities.**
- **It can be suggested by our direct sensory perception.**
- **The properties can be measured directly and easily.**

Microscopic Viewpoint

It is the viewpoint of the statistical description that deals with particles. Where a homogeneous substance can be visualized as being composed of an enormous number N of particles (atoms or molecules) having the same mass, and each of them is able to exist in a number of cases its energy is E_1, E_2, and E_3. The main issue is to find the number of particles in each energy state when reaching the equilibrium state.

This study is characterized by the following:

- **The assumptions take into account the composition of matter, such as the presence of particles.**
- **Several quantities must be measured.**
- **It can not be suggested by our direct sensory perception.**
- **These quantities can not be measured directly.**



- **The microscopic system consists of astronomical numbers of particles. If we apply Newton's laws to every particle in the system, we will reach an infinite number of equations to be solved, which take decades to be solved by computer. Even if we can find a solution for them after this period, we will find that the system has changed.**

- **Due to the smallness of atoms and molecules, many scientists, such as Maxwell and Boltzmann, have proposed using statistical methods to understand the relationship between the behavior of atoms and energy.**

- **It was noted that these methods were able to infer and predict thermal properties as expected by inductive methods. This trend is called statistical physics or thermodynamic statistics.**

❑ **The fundamental postulate of thermodynamic statistics:**

"*All possible microstates of an isolated group have equal probabilities.*"

➢ **Statistical Physics or Thermodynamic Statistics:**

It is a science which interested in studying (interpretation) the laws of thermodynamics (macroscopic properties of systems) by studying the microscopic properties of the systems.

Coin Model and the Most Probable Distribution

We start here with a simple experiment. We will extract some statistical terms that will help us in studding statistical physics. We start with throwing a single coin in the air. When the coin falls freely on the ground, we know in advance that we will get the event an image upwards and symbolize it with a letter *H*, Or the event is written up and we symbolize it by the letter *T* . The probability is 50% for either event, and this results from that

- **Assuming the probability of getting the event is a picture up is $P(H)$**
- **Assuming the probability of getting the event is a written up is $P(T)$**

Where, $P(H) + P(T) = 1$ **Then,** $P(H) = P(T) = \frac{1}{2}$

Symbol	Meaning
k	Numbering of cases
N_{1k}	The total number of cases the event obtained is an image (H) at the visual level k
N_{2k}	The total number of cases the event obtained is a written (T) at the visual level k
ω_k	Thermodynamic probability ≡ Number of microscopic cases k
$\Omega = \sum_{k=1} \omega_k$	The total number of microscopic cases
$P_k = \frac{\omega_k}{\Omega}$	The real probability

Let us study the results of rolling four identical but distinct coins ((for example, it can be distinguished by date or color).

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k	N	N	العملة1	العملة 2	العملة 3	العملة 4	ω	P
1	4	0	H	H	H	H	1	1/16
2	3	1	H	H	H	T	4	4/16
			H	H	T	H		
			H	T	H	H		
			T	H	H	H		
3	2	2	H	H	T	T	6	6/16
			T	T	H	H		
			H	T	H	T		
			T	H	T	H		
			H	T	T	H		
			T	H	H	T		
4	1	3	H	T	T	T	4	4/16
			T	H	T	T		
			T	T	H	T		
			T	T	T	H		
5	0	4	T	T	T	T	1	1/16

From the table we find that:

$$\Omega = \sum_{k=1}^{5} \omega_k = 1 + 4 + 6 + 4 + 1 = 16 \quad \rightarrow (1)$$

To calculate the average of getting the event *N,* we use the equation

$$\overline{N_j} = \frac{\sum_k N_{jk}\,\omega_k}{\Omega} = \sum_k N_{jk}\frac{\omega_k}{\Omega} = \sum_k N_{jk} P_k \quad \rightarrow (2)$$

We find the average getting the event an image (H)

$$\overline{N_1} = \frac{1}{16}[(4 \times 1) + (3 \times 4) + (2 \times 6) + (1 \times 4) + (0 \times 1)] = 2$$

- **Homework**

Calculate the average of getting the event written (T) and prove that $\overline{N_2} = 2$

In order for the results to be logical and reliable, we must deal with an infinite number of coins. And without tabulating the results, the expected values can be obtained. For a number of distinct coins N the number of ways to select a number of images N_1 and a number of written $(N - N_1)$ is given as:

$$\boxed{\omega_k = \binom{N}{N_1} = \frac{N!}{N_1!\,(N - N_1)!} \quad \rightarrow (4)}$$

From the previous table, we find that the highest rate of the number of microscopic cases of the ω_k distribution occurs when $N_1 = N_2 = 2$.

We call it $\tilde{\omega}$ which is called the most probable distribution

- **Homework: Check $\tilde{\omega}$ in the previous table using the equation(4)**

Example

Calculate $\tilde{\omega}$ for $N = 4, 8, 1000$

N	$N_1 = N/2$	$\tilde{\omega}$
٤	٢	$\frac{4!}{2!2!} = 6$
٨	٤	$\frac{8!}{4!4!} = 70$
١٠٠٠	٥٠٠	$\frac{1000!}{500!500!} \overset{?!}{=} 10^{300}$

Stirling's approximation

We know mathematically the factorial of numbers in the following form

$$n! = n \times (n-1) \times \dots \dots \times 3 \times 2 \times 1$$

And it is known: $1! = 0! = 1$

By taking the logarithm of equation (1)

$$\ln n! = \sum_{k=1}^{n} \ln k$$

And for a very large number, we can change the sum to an integral

$$\ln n! = \int_1^n \ln k \; dk \quad \rightarrow \quad \ln n! = [k \ln k - k]_1^n \approx n \ln n - n$$

Then we get

$$\boxed{\ln n! = n \ln n - n}$$

Which is known as the Sterling formula